Brown measure and invariant subspaces of operators

Brown measure is a sort of spectral distribution measure for elements of a finite von Neumann algebra and, more generally, for certain (unbounded) operators affiliated to a finite von Neumann algebra. This includes cases of random matrices. In a remarkable paper that was published in 2009, Uffe Haagerup and Hanne Schultz proved existence of hyperinvariant subspaces that decompose an operator belonging to a finite von Neumann algebra, the decomposition being according to Brown measure. A feature of the technique of the proof is use of a free perturbation of an operator in order to make mild the behaviour of the Brown measure.

These talks will cover the topics of invariant and hyperinvariant subspaces, Fuglede–Kadison determinants, Brown measure and Haagerup–Schultz projections. We will also discuss some joint work with Fedor Sukochev and Dmitriy Zanin, where, using Haagerup–Schultz projections we give an upper-triangular decomposition (like Schur's classical upper triangular form for matrices) for operators in finite von Neumann algebras. In the last talk, we will cover very recent work where we prove existence of Haagerup-Schultz projections and a Schur-type decomposition for certain unbounded operators affiliated to finite von Neumann algebras As an application, we show that certain "Dixmier traces" are spectral, i.e., their values on operators depend only on the Brown measures of the operators.

The talks will not assume knowledge of von Neumann algebras.